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TECHNICAL REPORT  
ERL-0367-TR

AUTOMATIC MODULATION RECOGNITION USING TIME DOMAIN PARAMETERS

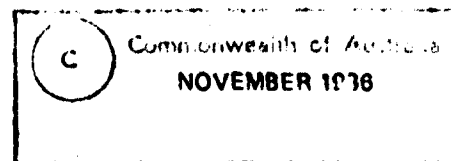
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AUTOMATIC MODULATION RECOGNITION USING TIME DOMAIN PARAMETERS

J. Aisbett

S U M M A R Y

One of the important variables to be determined when monitoring unknown radio transmissions is the modulation type. Automatic recognition procedures based on time domain parameters additional to the standard envelope and instantaneous frequency are proposed. Initial simulations which incorporate Gaussian noise perturbation and centre frequency and bandwidth mismatch support the theoretical arguments that the new parameters provide better modulation type discrimination in noisy conditions.



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## 1. INTRODUCTION

When unknown radio signals are monitored, either for military purposes or to enforce civilian compliance, one of the important variables of the transmission is modulation type(ref.1). The problem of replacing operator intervention by machine recognition has recently received attention. Pattern recognition procedures based on time-averaged behaviour of instantaneous envelope, frequency and zero-phase have been successfully applied to strong signals(ref.2 to 4).

Existing recognition procedures are less successful at low levels of signal-to-noise ratio (SNR), in part because these methods depend on parameters which, in the presence of bandlimited Gaussian noise, are biased estimators of the true signal parameters. Moreover, the standard approach to modulation recognition is inflexible in its explicit dealing with SNR. Typically, the signal envelope, frequency and zero phase are sampled at the Nyquist rate for a total duration of say, 0.1 to 1 s. The mean, standard deviation and, possibly, digitised distribution of each of these characteristics are computed. These values form a "feature vector" of the signal, which is compared with standard feature vectors of each of the modulation types being considered eg FM, AM, SSB, DSB, FSK. The classification of the signal is made according to some minimum distance criterion after some transformation of the space which attempts to weight components of the vector according to their discriminatory value. This ignores the intuitively obvious point that noisy signals are more similar to each other, regardless of modulation type, than they are to strong signals of the same modulation type.

In this preliminary report, we propose the use of a modulation recognition procedure based on time-domain signal parameters which are unbiased estimators of the true signal parameters in the presence of Gaussian noise with symmetric spectral density. Time averages and digitised distributions of these parameters would be used to form feature vectors, as outlined above. However, we propose assigning to each modulation type multiple classes, corresponding to various signal strengths.

In Section 2, we define the standard parameters and indicate theoretically why features based on them are unreliable discriminators of modulation type. In Section 3, we introduce some unbiased time domain parameters. In Section 4 we set up various modulation types, and in Section 5 describe the signals used in the limited simulations done so far. Results of those simulations, presented in Section 6, support the theoretical indications that the new approach enhances recognition of modulation types.

## 2. SIGNAL CHARACTERISTICS AND SNR

Let the received signal  $r(t)$  be

$$r(t) = A(t)\cos\theta(t) = A(t)\cos(\theta'(t)t + \delta(t)). \quad (1)$$

The decomposition of equation (1) can be made for any real-valued function  $r$  by defining the envelope  $A(t)$  and phase  $\theta(t)$  or instantaneous angular frequency  $\theta'(t)$  and zero phase  $\delta(t)$  in terms of  $r$ , its Hilbert transform  $\hat{r}$ ; and the time derivatives  $r'$  and  $\hat{r}'$ , viz:

$$\begin{aligned}
 A(t) &= (r(t)^2 + \hat{r}(t)^2)^{\frac{1}{2}} \\
 \theta(t) &= \arctan(r(t)/\hat{r}(t)) \\
 \theta'(t) &= (r(t)\hat{r}'(t) - \hat{r}(t)r'(t))/A^2(t) \\
 \delta(t) &= \theta(t) - \theta'(t)t.
 \end{aligned} \tag{2}$$

For narrowband signals on a relatively high frequency carrier, the Hilbert transform can be obtained instantaneously by analog means as in figure 1. In this case, the envelope and instantaneous frequency computed using equation (2) coincide with these signal characteristics as obtained directly by analog devices.

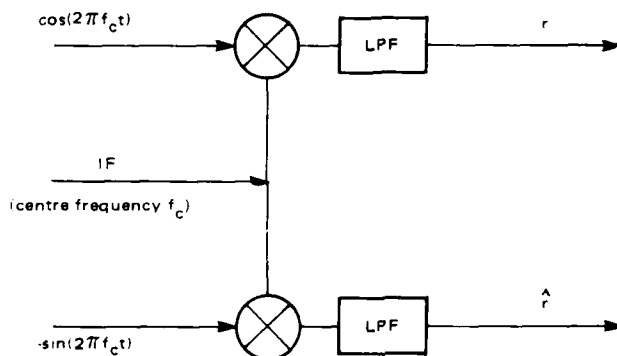


Figure 1. Analog computation of the Hilbert transform pair

Now suppose the received signal  $r(t)$  consists of a modulated signal  $S(t)$  with envelope  $B(t)$  and phase  $\phi(t)$ , plus a narrowband Gaussian noise  $n(t)$  of power  $\sigma^2$ . Then it is well known that the expected value,  $E(A(t))$ , of the envelope  $A(t)$  at any instant  $t$  is

$$E(A(t)) = \sqrt{\pi/2} \sigma M(-\frac{1}{2}, 1, -B^2(t)/2\sigma^2) \tag{3}$$

where  $M$  is the confluent hypergeometric function. (See for example reference 5, page 105). It is straightforward to show that  $E(A(t))$  increases monotonically with  $\sigma^2$ , having asymptotic expressions  $B(t) + 0(2\sigma^2/B(t))$  when  $\sigma^2 \ll B^2(t)$  and  $\sqrt{(\pi\sigma^2/2)} (1 + B^2(t)/4\sigma^2) + 0(B^4(t)/32\sigma^3)$  when  $\sigma^2 \gg B^2(t)$ . Similarly, for  $k \geq 1$ ,  $E(A^{2k}(t))$  increases monotonically with  $\sigma^2$ , from  $B^{2k}(t)$  at  $\sigma^2 = 0$  to the asymptote  $k!(2\sigma^2)^k$ . Specifically,

$$E(A^2(t)) = B^2(t) + 2\sigma^2 \tag{4}$$

and

$$E(A^4(t)) = B^4(t) + 8\sigma^2 B^2(t) + 2\sigma^4.$$

Rice's classic analysis of FM modulation showed that the instantaneous frequency is a biased estimator of the signal parameter in the presence of narrowband Gaussian noise with power spectrum symmetrically distributed about the centre frequency  $f_c$  (ref.6):

$$E(\theta'(t) - 2\pi f_c) = (1 - e^{-B^2(t)/2\sigma^2})(\phi'(t) - 2\pi f_c). \quad (5)$$

To get a feel for the distortion involved, consider a signal received with a SNR of 0 dB ie  $B(t) = \sqrt{2}\sigma^2$ . Then the expected value of  $\theta'(t) - 2\pi f_c$  in equation (5) is 63% of the true value and the expected value of  $A(t)$  in equation (3) is 128% of the true value. Trivially, at this SNR, the expected value of the received power is twice the true transmission power of any slowly-varying signal (cf equation (4)).

Signal modulation recognition also uses time-averaged sampled characteristics such as the variance  $\sigma_A^2$  of the signal envelope in the observation period T:

$$\sigma_A^2 = \frac{t}{T} \sum_{j=1}^n A^2(jt) - \left( \frac{t}{T} \sum_{j=1}^n A(jt) \right)^2 \quad \text{where } t = T/n. \quad (6)$$

Consider what happens to this variance if the signal strength falls so that  $|B(t)|$  is significantly less than  $\sigma$  for the entire observation period. The approximation  $E(A(t)) = \sqrt{\pi}\sigma^2/2 (1 + B^2(t)/4\sigma^2)$  discussed earlier is valid, so, using equation (4), the expected value of  $\sigma_A^2$  is

$$\begin{aligned} E(\sigma_A^2) &\approx \frac{t}{T} \sum_j (B^2(jt) + 2\sigma^2) - \frac{\pi}{4} (2\sigma^2 + \frac{t}{T} \sum_j (B^2(jt) + (\frac{t}{T} \sum_j (B^2(jt)))^2/8\sigma^2) \\ &\approx (1 - \frac{\pi}{4}) (\frac{t}{T} \sum_j B^2(jt) + 2\sigma^2). \end{aligned}$$

That is, the expected value of the variance of the envelope is proportional to the average power received. It is independent of the true envelope variance, and so carries no information on modulation type.

If the narrowband noise has power spectrum symmetrically distributed about the centre frequency, the second moment of  $\theta'(t)$  is infinite. (Although this is not a new result, it is not well documented and has been overlooked in at least one recent paper(ref.7), so a proof is given in Appendix I.) However, any least squares estimate of the second moment is of course finite. If the integrals in equation Appendix (I.7) are replaced by sampled estimates, the estimated expected value of  $(\theta' - 2\pi f_c)^2$  is a function of the transmitted signal's envelope B and its time derivative, and of the transmitted signal's instantaneous frequency  $\phi'$ . The exact dependence is complicated, but the

sensitivity of the second moment to transmitted signal parameters is manifested in another commonly-used time-averaged characteristic, viz., the variance of the signal instantaneous angular frequency over the observation period T:

$$\sigma_{\theta'}^2 = (t/T) \sum_j \theta'^2(jt) - ((t/T) \sum_j \theta'(jt))^2$$

To illustrate the behaviour of this characteristic with modulation type and noise, various signals modulated by a sinusoid of frequency 5 kHz were simulated with the procedure described in Section 5. The same Gaussian white noise was added to each signal, the total power was normalised, the signal was mixed down to baseband and then sampled at a frequency slightly greater than 10 kHz for 1024 samples. Instantaneous angular frequency was calculated as in equation (2). This was repeated at various signal to noise ratios. The value of  $\sigma_{\theta'}$  for each of the modulation types is recorded against SNR in Table 1.

The theoretical zero variance of the instantaneous frequency of AM, DSB, and CW signals is observed only at 100 dB. Again, the difficulty of using the variance as a modulation type discriminator without reference to SNR is apparent.

TABLE 1. STANDARD DEVIATION OF  $\theta'$  BY SNR BY MODULATION TYPE ( $\times 10$  kHz)

SNR (dB)	AM	DSB	SSB	CW	FM	
	(a)				(b)	(c)
100	0	0	0	0	.1	13
80	0	1	0	0	1	13
40	1	127	0	0	1	13
20	9	132	3	1	2	13
10	23	72	9	4	5	14
0	29	82	42	18	18	30
$-\infty$	42	42	42	42	42	42

(a) modulation depth 0.8

(b) modulation index 0.3

(c) modulation index 3

### 3. SOME ALTERNATIVE SIGNAL CHARACTERISTICS

The past use of the envelope, frequency and zero phase as basis for characterisation of signals needs no explanation. However, it is clear that characteristics which provide unbiased estimates in noise would be preferable, as long as they provide as good discrimination between the various modulation types in the absence of such noise. With this in mind, we put forward some estimators of various signal characteristics which are unbiased in the presence of narrowband Gaussian noise symmetrically distributed about the centre frequency  $f_c$ .



In the following, dependence on time of the various time functions is not made explicit. An overbar on a time function denotes the sampled time-average over the observation period T ie:

$$\bar{X} = \frac{1}{T} \sum_{j=1}^n X(jt), \quad T = nt.$$

Define  $\omega_c = 2\pi f_c$ ,  $\beta = \phi - \omega_c t$  and  $\gamma = \theta - \omega_c t$ .

### 3.1 Lemma

- (i)  $E(AA' = rr' + \hat{r}\hat{r}') = BB'$ .
- (ii)  $E(A^2\gamma' = rr' - r'r - \omega_c(r^2 + \hat{r}^2)) = B^2\beta'$ .
- (iii) If the noise is stationary over the observation period,  
 $E(\overline{A^2 - A'^2}) = \overline{B^2 - B'^2}$ .
- (iv) If the noise is stationary over the observation period,  
 $E(\overline{A^2\theta' - A'^2\theta'}) = \overline{B^2\phi' - B'^2\phi'}$ .

The proof of (iii) follows trivially from equation (4); (iv) follows from parts (ii) and (iii). Appendix II outlines the proofs of (i) and (ii).

Since the instantaneous values are unbiased, we can take as unbiased characteristics of the signal the digitised distribution of occurrences of values of the time functions  $A^2\gamma'$  and  $AA'$  about their time-based means.

Note that we are here talking only of the effects of noise, not of the variation between signals of the same modulation type with message.

We next discuss some time averaged signal parameters which could provide useful modulation type discriminators and which are unbiased over a sufficiently large number of observations, as long as the noise is stationary and the noise at different observations is uncorrelated (a standard assumption).

Define

$$\#(X,Y) = (2/n^2) \sum_{j=1}^n X(jt) \sum_{k=1}^n Y(kt) - \frac{1}{n} \sum_{j=1}^n X(jt)Y(jt) = 2 \bar{X} \cdot \bar{Y} - \overline{X \cdot Y}$$

That is, treating X and Y as functions of time  $\#(X,Y)$  is the product of their time-based means minus their time-based covariance.

## 3.2 Lemma

If  $n \gg 1/\text{SNR}$ , the following are unbiased estimators:

- (i)  $\#(A^2, A^2)$
- (ii)  $\#(A^2, AA')$
- (iii)  $\#(A^2, A^2(Y'+c))$ ,  $c$  any constant
- (iv)  $\#(AA', A^2(Y'+c))$ ,  $c$  any constant.

The proof of the lemma is sketched in Appendix II.

Part (i), which says that the square of the time based mean of the squared envelope minus its variance is an unbiased estimator, does not need the assumption of symmetry in the noise power spectrum. It can obviously also be expressed in terms of the time-based mean and variance of the signal power.

For a fixed amplitude signal (FM, CW etc) the time-based variance of  $B^2$  is zero, so that the expected value of Section 3.2(i) is  $B^2$ , and so is proportional to the square of the transmitted signal power, regardless of the total noise-plus-signal power received. This parameter is potentially extremely useful even with amplitude-varying signals. For instance, if a quaternary AM signal is observed over sufficient interval  $T$  to record a more-or-less equal number of the bit levels  $\pm b_o$ ,  $\pm 2b_o$ , then the time average of  $B^2$  is just  $\frac{1}{2}(b_o^2 + (2b_o)^2) = (5/2)b_o^2$  and of  $B^4$  is  $\frac{1}{2}(b_o^4 + (2b_o)^4) = (17/2)b_o^4$ . Hence  $\#(B^2, B^2) = 4b_o^4$ ; ie the expected value of the observable  $\#(A^2, A^2)$  indicates the true signal amplitudes.

## 4. THE SIGNAL MODELS

Consider a message signal  $m(t)$ , either digital or voice, transmitted on a carrier of frequency  $f$ . Write the transmitted signal  $S(t)$  as

$$S(t) = S_I(t)\cos(2\pi ft) - S_Q(t)\sin(2\pi ft). \quad (7)$$

Table 2 lists the pair  $(S_I, S_Q)$  up to a multiplicative factor according to modulation type: the lag  $\hat{t}$  allows for asynchrony between message and carrier.

TABLE 2. QUADRATURE PAIRS BY MODULATION TYPE

Modulation type	$S_I$	$S_Q$	Notes
AM	$1 + am(t + \ell)$	0	$a$ is modulation depth
DSB	$m(t + \ell)$	0	
SSB	$m(t + \ell)$	$\hat{m}(t + \ell)$	$\hat{\phantom{x}}$ denotes Hilbert transform
FM	$\cos(2\pi b \int_0^{t+\ell} m(\tau) d\tau)$	$\sin(2\pi b \int_0^{t+\ell} m(\tau) d\tau)$	$b$ is modulation index
FSK	$\cos(m(t + \ell)\Delta \cdot t)$	$\sin(m(t + \ell)\Delta \cdot t)$	$\Delta/2\pi$ is frequency jump
PSK	$\cos(\pi m(t + \ell)/(a+1))$	$\sin(\pi m(t + \ell)/(a+1))$	data set $\{\pm 1, \dots, \pm a\}$ .

We assume that the transmitted signal essentially lies in the band  $f \pm B_m$ ; that is, for the amplitude modulated signals or narrowband FM signals, we require  $m(t)$  to have bandwidth at most  $B_m$ , whilst for wideband FM, we require this bandwidth to be much less than the modulation index  $b$  where  $b \leq B_m$ . For the digital cases PSK and FSK, we must impose conditions on the pulse shape to meet this bandwidth restriction. With  $S(t)$  thus constrained, its Hilbert transform  $\hat{S}(t)$  is simply

$$\hat{S}(t) = S_Q \cos(2\pi ft) + S_I \sin(2\pi ft). \quad (8)$$

Next, suppose the incoming signal  $r(t)$  is received in the band  $f+d \pm B$ , where  $d$  is the offset from accurate tuning on the signal, and is assumed to satisfy  $|d| < B - B_m$  so that the transmitted signal lies within the received band. The incoming signal is the sum of the transmitted signal  $S(t)$  (passed without distortion), and a noise component  $n(t)$  which we take to be stationary, Gaussian and with power spectral density symmetrically distributed about  $f+d$ . This narrowband noise is described by

$$\begin{aligned} n(t) &= n_1(t) \cos(2\pi(f+d)t) - n_2(t) \sin(2\pi(f+d)t) \\ &= p_1(t) \cos(2\pi ft) - p_2(t) \sin(2\pi ft) \end{aligned} \quad (9)$$

where

$$p_1(t) = n_1(t) \cos(2\pi dt) + n_2(t) \sin(2\pi dt)$$

and

$$p_2(t) = -n_2(t) \cos(2\pi dt) + n_1(t) \sin(2\pi dt).$$

Its Hilbert transform is  $\hat{n}(t) = p_2(t) \cdot \cos(2\pi ft) + p_1(t) \cdot \sin(2\pi ft)$ . The set  $\{n_1(t), n_2(t)\}$  consists of observations from identically distributed Gaussian random variables with variance equal to the average power of  $S(t)$  divided by the signal-to-noise ratio SNR. With a rectangular or Gaussian bandpass filter acting on white noise we can suppose the observations are from independent random variables when sampled at the Nyquist rate.

Finally, we suppose that in obtaining the Hilbert transforms (cf figure 1) the signals are mixed back to baseband modulo the tuning offset and a constant phase offset  $\phi$ . That is, we set  $f+d = 0$ .

## 5. SIMULATING THE INFORMATION SIGNALS AND NOISE

Voice message signals  $m(t)$  were obtained via a 64 kHz digitisation of male speech. The derivative  $m'$  and the integral  $\int_0^t m$  were obtained numerically at

this sampling rate. The resulting data were subsequently sampled at 8 kHz for AM and DSB modulation, and 32 kHz for wideband FM. An example of this voice data is shown in figure 2(a).

Digital message signals with user defined symbol duration  $T$  were generated by randomly selecting a "data" sequence  $a_0, a_1, a_2 \dots$  from the set  $\{\pm 1, \pm 2, \dots, \pm a\}$  and putting this information on a Gaussian pulse of bandwidth  $B_m$  truncated to a 3-symbol duration. Specifically, setting  $k = [t/T]$ ,  $\Delta = t - kT$ , we sampled  $m(t)$  defined for  $kT \leq t < (k+1)T$ ,  $k \geq 1$ , by

$$m(t) = \frac{1}{a} (a_{k-1} e^{-2\pi B_m^2 (\Delta+T/2)^2} + a_k e^{-2\pi B_m^2 (\Delta-T/2)^2} + a_{k+1} e^{-2\pi B_m^2 (\Delta+3T/2)^2}). \quad (10)$$

The message bandwidth  $B_m$  and sampling rate  $2B_m$  were user-selected; to avoid excessive interference between adjacent symbols, only  $B_m T$  products greater than 1 were selected. Figure 2(b) is an example of binary message data. The sampled derivative  $m'$  was approximated by the analytic derivative of equation (10), and the integral of  $m$  was obtained numerically.

The Hilbert transform of the message signals was not computed, so that SSB signals were not simulated.

The message signals were applied to the various modulation types as in §3, to generate the transmitted signals  $S(t)$ . The same noise  $n(t)$  was added to each, to give received signals  $r(t)$ . The inphase and quadrature noise observations  $n_1(t)$  and  $n_2(t)$  were modelled by the returns from a Gaussian random number generator with variance  $1/(1+\text{SNR})$  where SNR is the user-supplied signal-to-noise ratio. The average received signal power was approximately set to 1 by normalising  $S(t)$  digitally so as to have average power  $\text{SNR}/(1+\text{SNR})$ . For example, with AM, the pair  $(S_I, S_Q)$  of Table 1 was divided by

$$\frac{SNR + 1}{2 n SNR} \sum_{i=1}^n (1 + a.m(t_i))^2)^{\frac{1}{2}},$$

where  $m(t_i)$  was the  $i^{th}$  sample of the message signal ie  $m(t_i) = m((i+\epsilon)/2B)$  for  $\epsilon$  a user-defined lag. The number of samples  $n$  was chosen to give a total observation period of about 90 ms.

The Hilbert transform  $\hat{r}(t)$  was readily computed from the equation

$$r(t) = (S_I(t) + p_1(t)) \cos(2\pi dt + \phi) - (S_Q(t) + p_2(t)) \sin(2\pi dt + \phi). \quad (11)$$

Since the functions  $m'(t)$  needed to compute  $S_I'(t)$  and  $S_Q'(t)$  were supplied, the derivatives  $r'(t)$  and  $\hat{r}'(t)$  were also readily computed, given the derivatives  $n_1'(t)$  and  $n_2'(t)$  needed to compute  $p_1(t)$  and  $p_2(t)$ . These were obtained from a Gaussian random number generator whose variance depended on the bandpass filter modelled: the variance was  $\pi^2 B^2 / (6(1+SNR))$  for a rectangular filter, or  $4\pi^2 B^2 / (1+SNR)$  for a Gaussian filter.

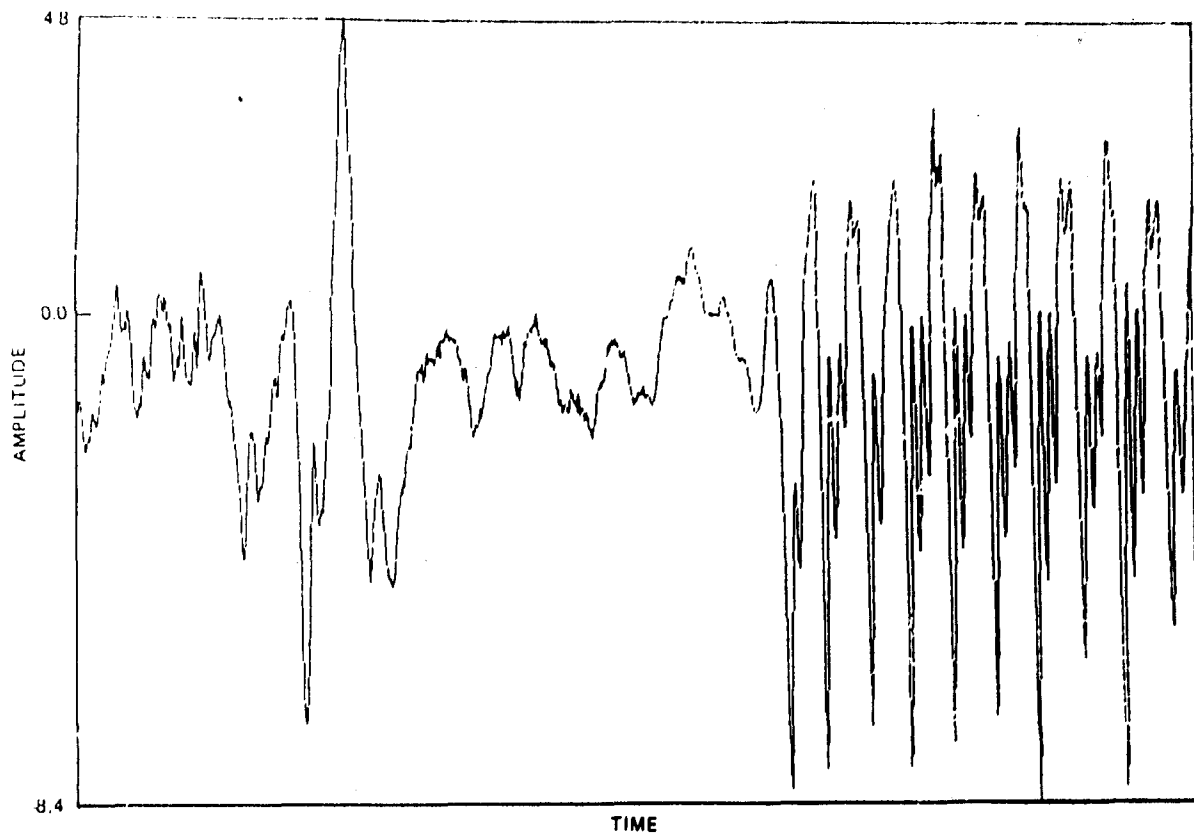
## 6. SIMULATION RESULTS

Figures 3 to 7 show the digitised distributions of the standard variables (the envelope  $A$  and the normalised instantaneous angular frequency  $\hat{\omega}'$ ) about their mean values, plus the unbiased distributions of  $A^2$ ,  $AA'$  and  $A^2\hat{\omega}'$  about their means. Each plot is the average of 10 runs, from 3 or 4 different modulating signals at 1 to 3 different SNR levels. The SNR levels were in the range 15 to 30 dB for strong signals (group (a) figures), and -1 to 3 dB for weak signals (group (b) figures). Group A figures are from voice-modulated AM, DSB and wide- and narrowband FM signals, and group B figures are from binary AM, DSB and narrowband FM signals. The corresponding distribution for the Gaussian white noise is included in all figures for comparison. CW signals are included with group B for convenience of display. The marked visual difference between group (a) and (b) figures make a strong heuristic argument for the inclusion into any classification scheme of subclasses related to the SNR.

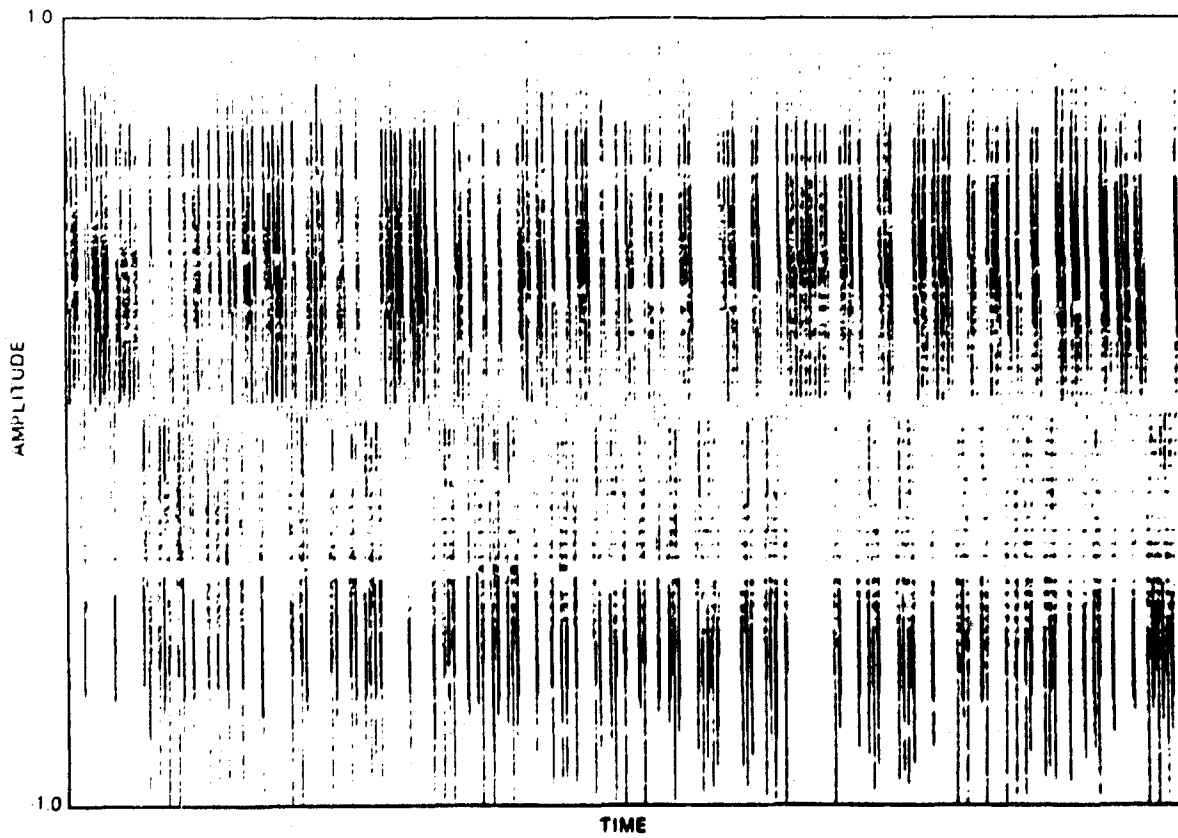
We will discuss each of these figures in turn, but caution that one cannot fully analyse application of the parameters discussed in the preceding sections on so few signals and modulating forms. Rather, this section indicates the relative performance of the various parameters.

In the following, the parameter features used to make comparisons are:

- (i) the number of occurrences of the most commonly observed quantised distance from the mean: that is, the peak of the distribution and



(a) Voice data



(b) Binary data

Figure 2. Examples of message signals

(11) the number of occurrences of distances greater or less than prespecified limits: that is, the tails of the distribution.

The 2-sided t-test is used to determine whether mean values recorded for the various modulation types are the same within a 99% confidence level. (Here, the mean value referred to is the average value of the histogram bin recorded over the 10 simulation runs. In using this test, we are implicitly assuming that the number of observations which lie in a particular histogram bin is a normally distributed random variable with variance independent of the modulation type. However, the t-test is known to be robust.) We use standard, albeit statistically loose, terminology below in saying that "two modulation types can be distinguished" when the t-test fails, and by saying "the values are the same" if it is passed.

(1) Figure 3: Distribution of envelope A about mean

In figure 3, the envelope of the noise approximates the Rayleigh distribution which in the large sample limit it must follow. The envelope of a DSB signal is the absolute value of the modulating signal. As a consequence of the randomness essential to any information-carrying signal, the distribution of the voice DSB envelope is very similar to the Rayleigh distribution for the voice signal (figure 3A(a)). The envelope of the AM signal bears the influence not only of the message but also of the carrier, so for strong signals has a higher peak at the mean. Finally, the FM and CW strong transmissions, with their almost constant envelopes, have sharply spiked distributions.

However, for weak signals, ALL the modulation types have the same distribution of envelope as has the white noise. (Figure 3A(b) and 3B(b)). Modulation discrimination is unreliable using these distributions, as foreshadowed in Section 3.

(2) Figure 4: Distribution of instantaneous angular frequency  $\theta'$  about mean

In figure 4, the distribution for noise of the instantaneous frequency  $\theta'/2\pi$  about its mean approximates a density function of the form  $a(1+\theta'^2/b)^{-3/2}$  (see Appendix III). For strong voice signals (figure 4A(a)), the narrowband FM distribution follows a noise-like distribution, again reflecting the randomness of the modulating signal.

In line with the sort of results recorded in Table 1 and discussed in Section 3, the distribution for strong CW signals appears sharply peaked, but approaches that of the noise when the signal is weak (figure 4B). Since the strong signal category includes SNR's down to 15 dB, there is sufficient noise to broadly spread the mean DSB distributions (figures 4A(a) and 4B(a)), although as with the CW signals there is a large variance associated to each bin value. On the 10 runs performed on strong CW signals, the mean histogram values returned were not significantly different from zero - with a one-sided t-test, 99% confidence level - or from the corresponding strong voice FM histogram values. Although figure 4A(a) suggests a moderately peaked distribution for the strong AM voice signals, none of the histogram values were in fact significantly different from the corresponding white noise, weak FM, CW or strong digital FM values. The remaining AM and DSB signal types (strong or weak, voice or binary) could not be distinguished. Only the strong voice, FM signals could be separated from the various FM signal types.

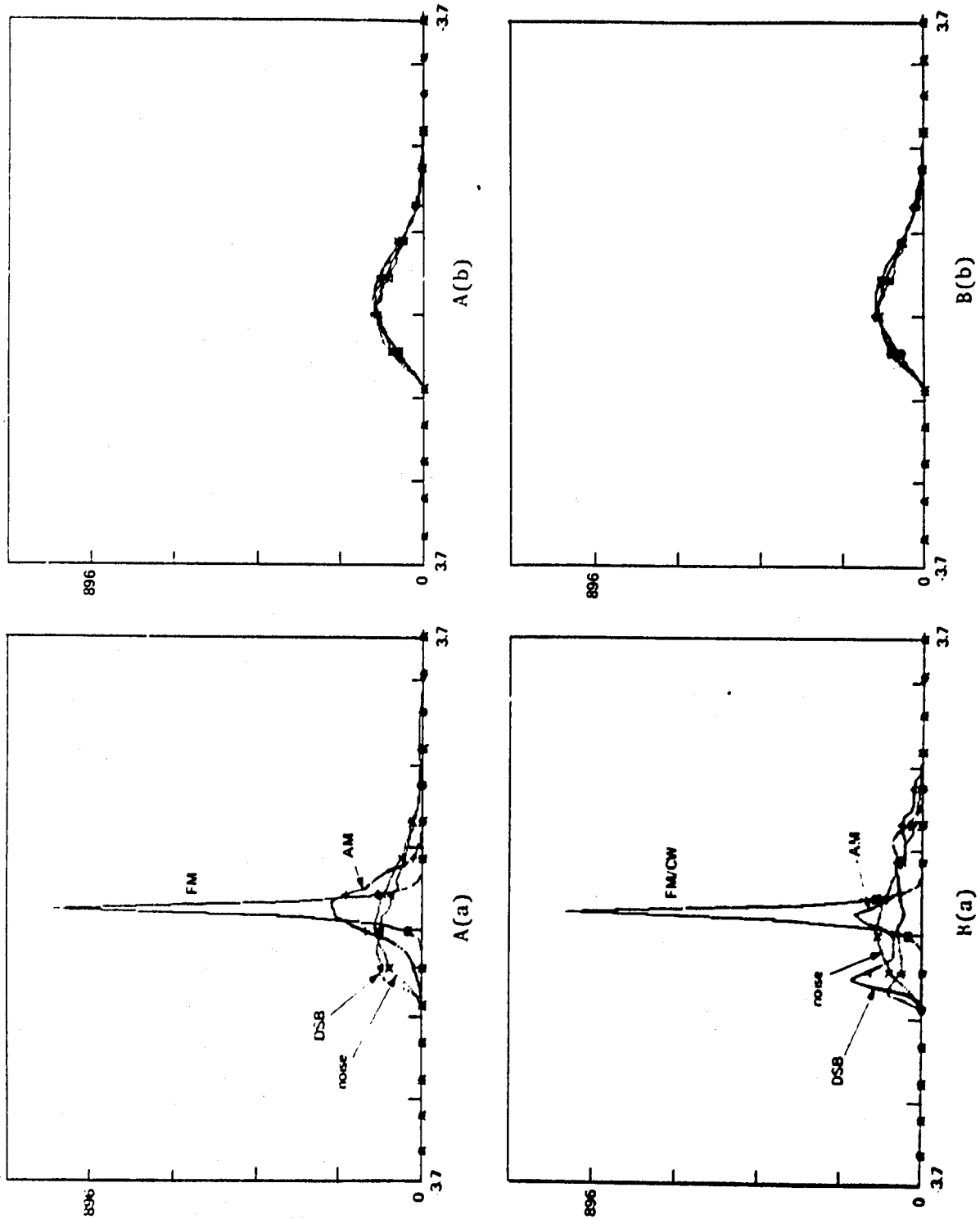


Figure 3. Distribution of envelope A about mean



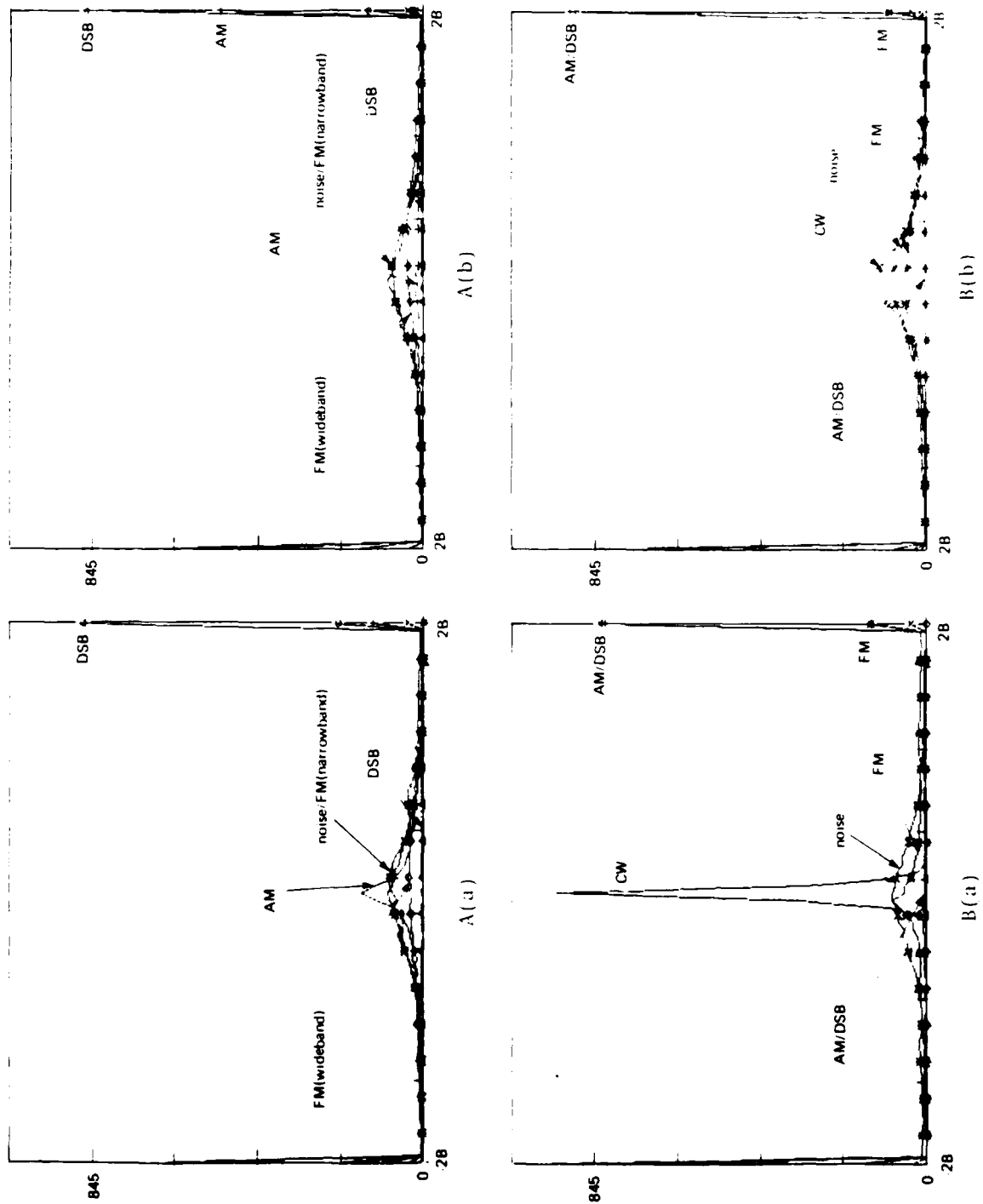


Figure 4. Distribution of instantaneous angular frequency  $0'$  about mean

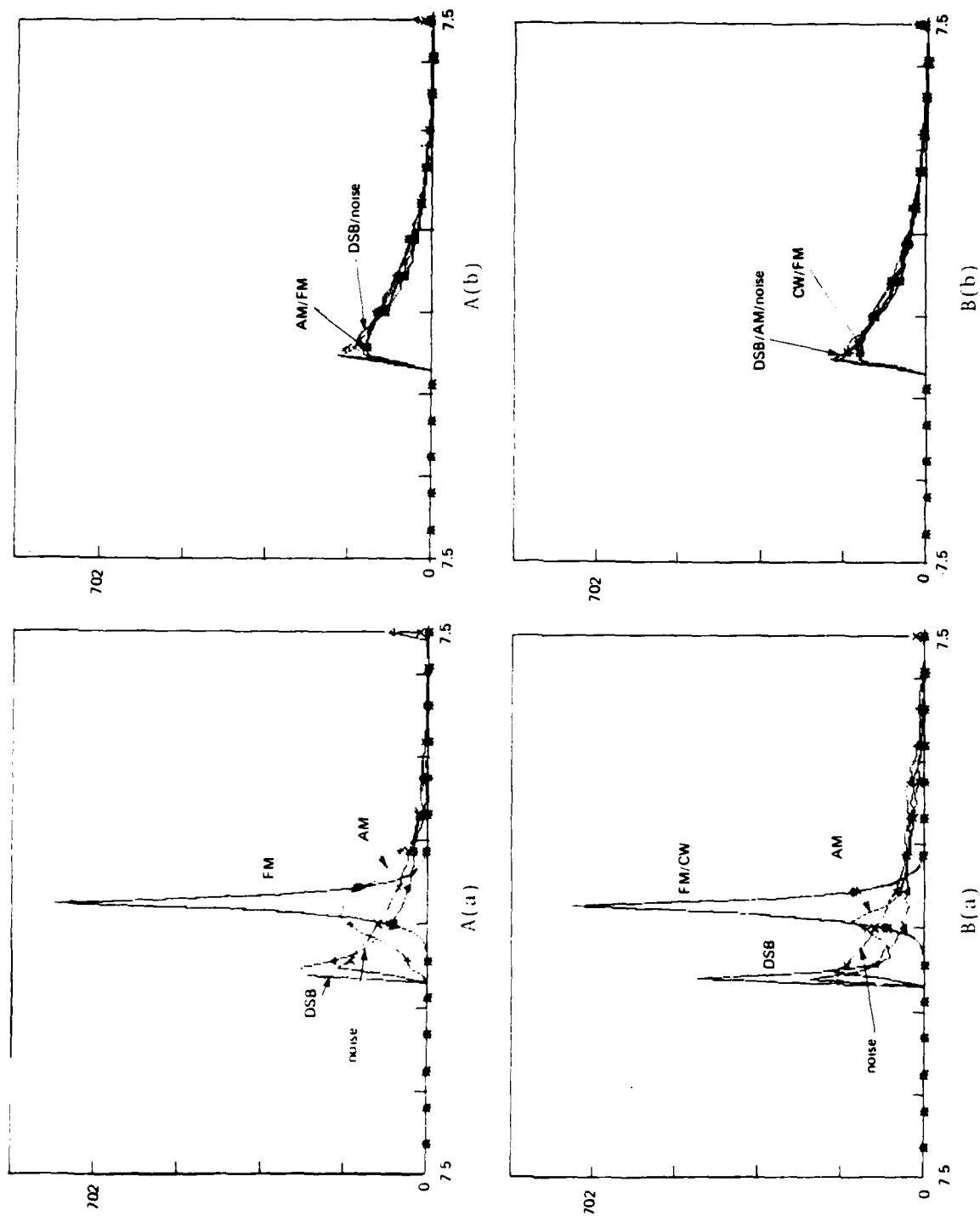


Figure 5. Distribution of  $A^2$  about mean

(3) Figure 5: Distribution of  $A^2$  about mean

In the large sample limit, the distribution of the square of the envelope of white noise is the exponential distribution(ref.5). Shifted by the mean, it has asymptote at -2, because the average value of the envelope squared is twice the average receiver power which we have normalised to unity. Comments made in Subsection (1) concerning the envelope generally apply here, with the important exception that for noisy signals, only the digital DSB cannot be statistically distinguished from noise.

(4) Figure 6: Distribution of  $AA'$  about mean

The variable  $AA'$  is proportional to the derivative of the envelope squared. For the white noise considered, it is the sum of products of independent Gaussian random processes (cf. 3.1(i)) and so is itself normally distributed. This is observed in figure 6.

Visually, three groups can be recognised for the strong signals - AM/DSB, wideband FM/noise, and narrowband FM/CW (figures 6A(a) and 6B(a)). However, statistically the FM, CW and noise distributions coincide and voice AM, binary AM, and DSB are mutually distinguishable. The addition of noise (figure 6(b)) does not significantly alter the distributions of digital DSB and AM but for the voice data spreads the AM distribution to make it significantly different to all other signal types considered. This is also the case for the wideband FM distribution. The remaining FM signal types, CW and noise could not be statistically distinguished.

(5) Figure 7: Distribution of  $A^2\theta'$  about mean

Following equations (2), the variable  $A^2\theta'$  can be expressed as a difference of products which, for the noise signal considered here, are Gaussian random processes. Hence, as observed in figure 7, the noise distribution is approximately normal. For the strong CW signal, it is sharply peaked (figure 7B(a)).

Comments made about the distribution of instantaneous frequency (Subsection (2)) generally apply, except, predictably, more discrimination between the various AM and DSB signal types is possible.

As a comparative summary of the results obtained, Tables 3 and 4 display symmetric binary matrices, where a zero in the (i,j)th position indicates that all features (mean histogram values) in the set under consideration were the same for the ith and jth signal types (t-test, 99% confidence limit). The features are, for Table 3, the peak and tails values of the histograms of the envelope  $A$  and instantaneous angular frequency  $\theta'$  and, for Table 4, the peak and left tail value of the histograms of  $A^2$ ,  $AA'$ , and  $A^2\theta'$  (a total of six features in each case). Using the standard parameters there are 14 unresolved signal pairs, whilst with the new parameters there are only two.

Table 5 lists, for the various modulation types and for strong and weak signals, the mean and the associated standard deviation over the 10 simulation runs of the variable  $\#(A^2, A^2)$  defined in Section 3.2(i). As expected, this parameter appears to be a very good way of determining the true signal power, and hence the SNR, of constant envelope signals. The other variables defined in Section 3.2 proved to have large standard deviations within each signal type, and so appear to have limited use in modulation recognition. However, this very variability may make them useful fine tuners, for instance, in distinguishing between voice and digital modulating signals, given an estimate of SNR.

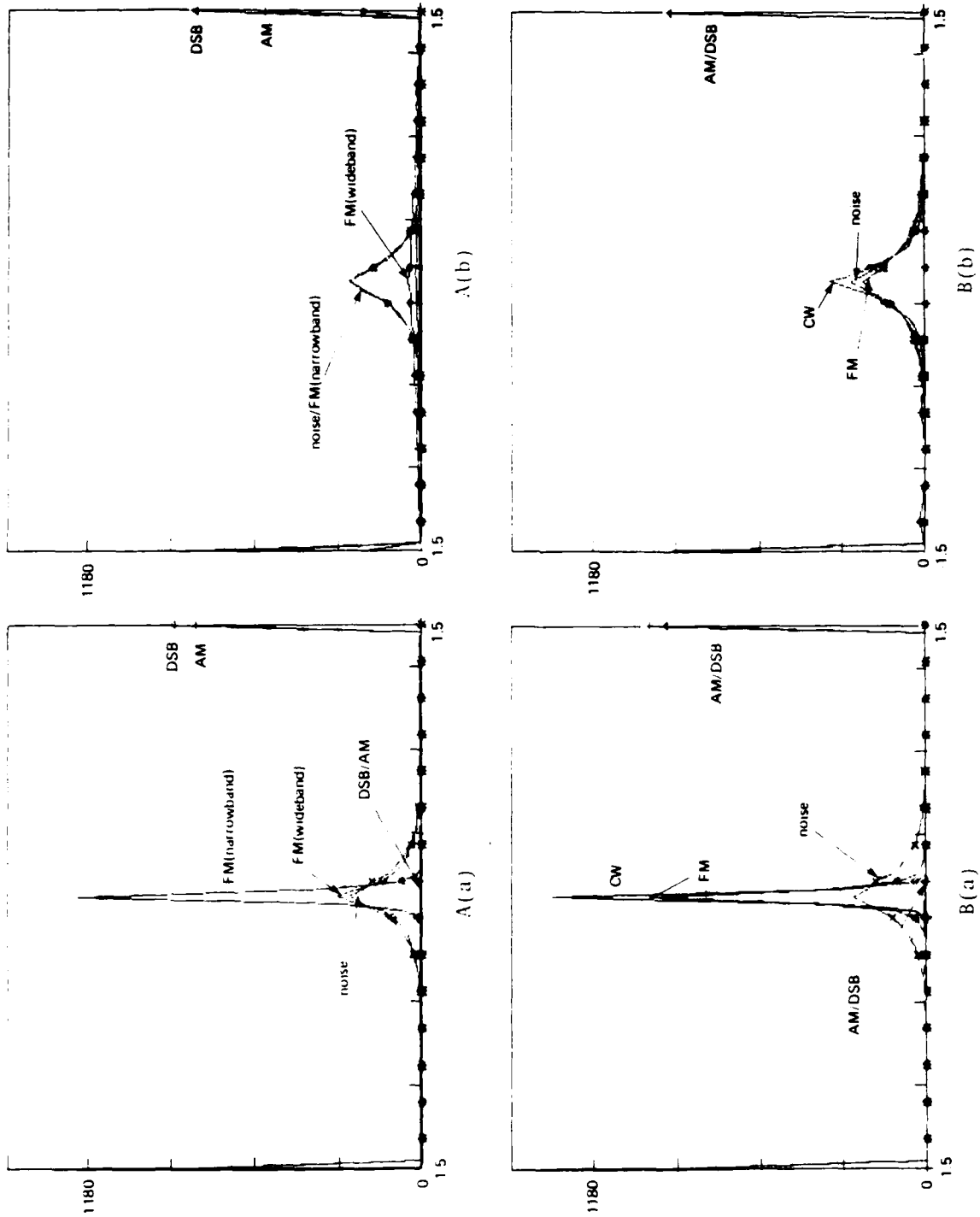


Figure 6. Distribution of AA' about mean

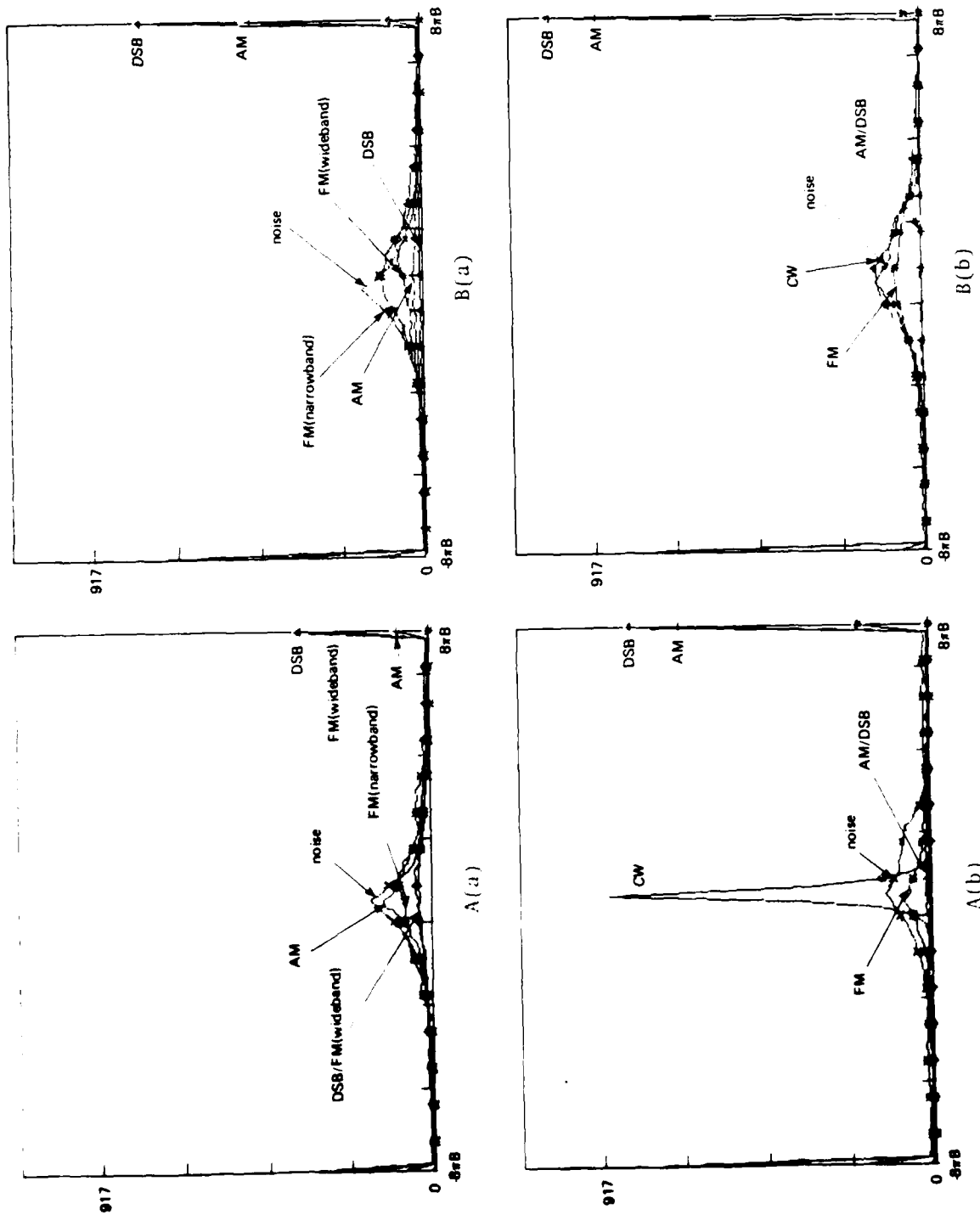


Figure 7. Distribution of  $A^2\theta'$  about mean

TABLE 3. SIGNIFICANTLY DIFFERENT FEATURE VALUES BY SIGNAL TYPE (A,  $\theta'$ )  
(t-TEST, 99% CONFIDENCE LEVEL)

Features: peak and tails of distributions of envelope A and  
instantaneous angular frequency  $\theta'$ .

	Strong voice-Aa				Weak voice-Ab				Strong binary-Ba				Weak binary-Bd				
	AM	DSB	FM nb	FM wb	AM	DSB	FM nb	FM wb	AM	DSB	FM nb	CW	AM	DSB nb	FM	CW	Noise
AM	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
DSB	1	0	1	1	0	0	1	1	1	0	1	1	0	0	1	1	1
Aa FM-nb	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FM-wb	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
AM	1	0	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1
DSB	1	0	1	1	1	0	1	1	1	0	1	1	0	0	1	1	1
Ab FM-nb	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
FM-wb	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
AM	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1
DSB	1	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1
Ba FM	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
CW	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
AM	1	0	1	1	1	0	1	1	1	0	1	1	0	0	1	1	1
DSB	1	0	1	1	1	0	1	1	1	0	1	1	0	0	1	1	1
Bb FM-nb	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
CW	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
Noise	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

TABLE 4. SIGNIFICANTLY DIFFERENT FEATURE VALUES BY SIGNAL TYPE  
( $A^2$ ,  $AA'$ ,  $A^2\theta'$ ) (t-TEST, 99% CONFIDENCE LEVEL)

Features: peak and left tail of distributions of  $A^2$ ,  
 $AA'$  and  $A^2\theta'$

	Strong voice-Aa				Weak voice-Ab				Strong binary-Ba				Weak binary-Bb				
	AM	DSB	FM nb	FM wb	AM	DSB	FM nb	FM wb	AM	DSB	FM nb	CW	AM	DSB nb	FM	CW	Noise
AM	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
DSB	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Aa FM-nb	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
FM-wb	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
AM	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
DSB	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1
Ab FM-nb	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
FM-wb	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
AM	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
DSB	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
Ba FM-nb	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
CW	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
AM	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1	1	1
DSB	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
Bb FM	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
CW	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
Noise	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

nb - narrowband

wb - wideband

TABLE 5. THE MEAN-SQUARED-ENVELOPE SQUARED MINUS ITS VARIANCE  $\#(A^2, A^2)$

	AM		DSB		Narrowband FM		Wideband FM	CW	Noise
	(a)	(b)	(a)	(b)	(a)	(b)	(a)		
Strong signals									
mean	3.1	1.0	-4.0	-1.0	3.9	3.9	3.9	3.8	0.11
sd	0.3	0.7	1.4	2.1	0.1	0.1	0.1	0.1	0.16
Weak signals									
mean	1.0	0.3	-1.1	-0.4	1.2	1.1	1.2	1.1	
sd	0.3	0.3	0.7	0.6	0.4	0.4	0.4	0.4	

(a) voice modulated signal;  
(b) digitally modulated signal

## 7. CONCLUSIONS

We have shown theoretically why features based on the standard time domain parameters of signal envelope and instantaneous frequency are strongly influenced not only by the modulation type but also by the signal to noise ratio. Parameters which are unbiased in the presence of Gaussian white noise have been introduced and limited simulations run to compare their performances with those of the standard parameters.

Discrimination between transmissions of modulation type AM, FM, DSB or CW on the basis of characteristics of the distribution of the new parameters appears to be at least as good on strong signals, while in a noisy environment modulation recognition is markedly enhanced. The further discrimination between voice and binary data using these techniques may also be feasible. More work on the statistics of the speech and multilevel digital modulating signals needs to be done. Performance in non-Gaussian noise also needs to be evaluated both theoretically and practically.

Further simulations, on a much larger data base, and extensions to SSB, OOK, PSK and FSK are planned or under way. A hardware preprocessor is being built to provide analytic computation of the new variables, which, after digitisation, will aid in providing a sufficiently large signal set for the implementation of a classification scheme. It is envisaged that the Fisher linear discriminant function will be applied to features obtained from the parameter distributions.

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# APPENDIX I

## VARIANCE OF THE INSTANTANEOUS FREQUENCY

Suppose a received signal  $r(t) = A(t)\cos(\theta(t))$  is the sum of a transmitted signal  $S(t) = B(t)\cos(\phi(t))$  and narrowband Gaussian noise  $n(t) = n_1(t)\cos(\omega_c t) - n_2(t)\sin(\omega_c t)$  which has power spectrum symmetrically distributed about the centre frequency  $\omega_c/2\pi$ . Then the expected value of  $\theta'^2$  is not defined (is infinite).

Proof. Let  $\gamma = \theta - \omega_c t$  and  $\beta = \phi - \omega_c t$ . With respect to the centre frequency,  $r$  has inphase component  $R_I = B\cos\beta + n_1$ , and quadrature component  $R_Q = B\sin\beta + n_2$ . It is straightforward to compute

$$r' = R_I' \cos\omega_c t - R_Q' \sin\omega_c t - \omega_c \hat{r}$$

and

$$\hat{r}' = R_Q' \cos\omega_c t + R_I' \sin\omega_c t + \omega_c r. \quad (I.1)$$

Moreover (cf equation (2)),

$$\gamma' \equiv \theta' - \omega_c = (r(\hat{r}' - \omega_c r) - \hat{r}(r' + \omega_c \hat{r})) / (r^2 + \hat{r}^2). \quad (I.2)$$

From equations (I.1) and (I.2), we get

$$\gamma' = (R_I R_Q' - R_I' R_Q) / (R_I^2 + R_Q^2). \quad (I.3)$$

Substituting the definitions of  $R_I$  and  $R_Q$  gives

$$\begin{aligned} \gamma' = & \{ (B\cos\beta + n_1)(B'\sin\beta + \beta'B\cos\beta + n_2') - (B'\cos\beta - \beta'B\sin\beta + n_1')(B\sin\beta + n_2) \} \\ & / \{ (B\cos\beta + n_1)^2 + (B\sin\beta + n_2)^2 \}. \end{aligned} \quad (I.4)$$

Recall that, at any instant  $t$ ,  $n_1$ ,  $n_2$ ,  $n_1'$  and  $n_2'$  are independent random variables with zero mean. Moreover,  $E(n_1^2) = E(n_2^2) = \sigma^2$ , and  $E(n_1'^2) = E(n_2'^2)$ . (This is a consequence of our assumptions on the form of noise.) It is then easy to show from equation (I.4) that

$$\begin{aligned}
E(\gamma'^2) &= E\{[(B'\sin\beta + \beta'B\cos\beta)(B\cos\beta + n_1) - (B'\cos\beta - \beta'B\sin\beta)(B\sin\beta + n_2)]^2 \\
&\quad / [(B\cos\beta + n_1)^2 + (B\sin\beta + n_2)^2]\} \\
&\quad + E(n_1'^2) \cdot E(1/[(B\cos\beta + n_1)^2 + (B\sin\beta + n_2)^2]).
\end{aligned} \tag{I.5}$$

Define  $\alpha = \gamma - \beta$ , so that  $n_1 + B\cos\beta = A\cos(\alpha + \beta)$  and  $n_2 + B\sin\beta = A\sin(\alpha + \beta)$ . A short calculation then reduces equation (I.5) to

$$E(\gamma'^2) = E((-B'\sin\alpha + \beta'B\cos\alpha)^2/A^2) + E(n_1'^2) E(1/A^2). \tag{I.6}$$

Now because  $n_1$  and  $n_2$  are independent Gaussian random variables, they have joint density function

$$p(n_1, n_2) = (1/2\pi\sigma^2)^{-1} \exp(-(n_1^2 + n_2^2)/2\sigma^2) = (1/2\pi\sigma^2)^{-1} \exp(-(A^2 + B^2 - 2AB\cos\alpha)/2\sigma^2). \tag{I.7}$$

From the Jacobian calculation, we have  $dn_1 dn_2 = A dA d\alpha$ . Hence equation (I.6) becomes

$$\begin{aligned}
E(\gamma'^2) &= (1/2\pi\sigma^2)^{-1} e^{-B^2/2\sigma^2} \int_0^{2\pi} d\alpha ((-B'\sin\alpha + \beta'B\cos\alpha)^2 + E(n_1'^2)) \cdot \\
&\quad \int_0^\infty dA (e^{-(A^2 - 2AB\cos\alpha)/2\sigma^2}) / A.
\end{aligned} \tag{I.8}$$

Set  $I_A(\delta) = \int_0^\infty dA (e^{-(A^2 - 2AB\cos\alpha)/2\sigma^2}) / A$ . Clearly  $I_A(\delta)$  does not converge as  $\delta \rightarrow 0$ . But from equation (I.8), if  $\delta \geq 0$ ,

$$E(\gamma'^2) > (e^{-B^2/2\sigma^2}/(2\pi\sigma^2)) I_A(\delta) \left( \int_0^{2\pi} d\alpha (-B'\sin\alpha + \beta'B\cos\alpha)^2 + 2\pi E(n_1'^2) \right).$$

This implies our result.

## APPENDIX II

### SOME UNBIASED ESTIMATORS

To prove that the functions defined in Lemmas 3.1 and 3.2 are unbiased estimators as claimed, one can proceed as follows.

We have

$$\begin{aligned} r(t) &= S(t) + n(t), \\ \hat{r}(t) &= \hat{S}(t) + \hat{n}(t) \end{aligned} \quad (\text{II.1})$$

where  $r = A \cos \theta = A \cos(\gamma + \omega_c t)$  and  $S = B \cos \phi = B \cos(\beta + \omega_c t)$ .

The noise  $n$  has input power  $\sigma^2$ . If  $n_I$  and  $n_Q$  are respectively its inphase and quadrature components with respect to the centre frequency  $f_c$ , then  $n_I, n_Q$  and their time derivatives  $n'_I$  and  $n'_Q$  are (at any instant  $t$ ) independent zero-mean Gaussian random variables. The variance of  $n_I$  and  $n_Q$  is  $\sigma^2$ , and the variance of  $n'_I$  and  $n'_Q$  is  $C(B)\sigma^2$ , where the form of the function  $C$  of noise bandwidth  $B$  is determined by the noise power spectrum shape. Now

$$\begin{aligned} n' &= n'_I \cos \omega_c t - n'_Q \sin \omega_c t - \omega_c \hat{n} \\ \hat{n}' &= n'_Q \cos \omega_c t + n'_I \sin \omega_c t + \omega_c n. \end{aligned} \quad (\text{II.2})$$

Using equation (II.2), and the independence of the random variables  $n_I, n'_I, n_Q$  and  $n'_Q$ , it is straightforward to compute

$$\begin{aligned} E(\hat{n}\hat{n}) &= E(n_I n_Q (\cos^2 \omega_c t - \sin^2 \omega_c t) + (n_I^2 - n_Q^2) \cos \omega_c t \sin \omega_c t) = 0; \\ E(\hat{n}n') &= 0 = E(\hat{n}\hat{n}'); \\ E(\hat{n}\hat{n}') &= -\omega_c E(\hat{n}^2) = -\omega_c \sigma^2; \\ E(\hat{n}n') &= \omega_c \sigma^2; \\ E(n'^2) &= C(B)\sigma^2 + \omega_c^2 \sigma^2 = E(\hat{n}'^2). \end{aligned} \quad (\text{II.3})$$

Finally, we will need the fact that if  $u_1, u_2, u_3$  and  $u_4$  are zero mean Gaussian random variables, then  $E(u_1 u_2 u_3) = 0$  and  $E(u_1 u_2 u_3 u_4) = E(u_1 u_2)E(u_3 u_4) + E(u_1 u_3)E(u_2 u_4) + E(u_1 u_4)E(u_2 u_3)$ , (reference 5, page 97).

To prove Section 3.1(ii) viz;  $E(A^2\gamma') = B^2\beta'$ , use the definitions in equation (2), to write

$$A^2\gamma' = (\hat{r}\hat{r}' - \hat{r}\hat{r}') - \omega_c(\hat{r}^2 + \hat{r}'^2) = \hat{r}(\hat{r}' - \omega_c \hat{r}) - \hat{r}'(\hat{r} + \omega_c \hat{r}').$$

Rewriting as in equation (II.1), this is

$$\begin{aligned} & (S+n)(\hat{S}' + \hat{n}' - \omega_c(S+n)) - (\hat{S} + \hat{n})(S' + n' + \omega_c(\hat{S} + \hat{n})) \\ &= S(\hat{S}' - \omega_c S) - \hat{S}(S' + \omega_c \hat{S}) + \hat{n}\hat{n}' - \omega_c n^2 - \hat{n}\hat{n}' - \omega_c \hat{n}^2 + \text{terms linear in noise.} \end{aligned}$$

Using equation (II.3)  $E(\hat{n}\hat{n}' - \hat{n}\hat{n}') = \omega_c E(n^2 + \hat{n}^2)$ , and of course, the expected value of any term linear in noise (ie in  $n, \hat{n}, n'$  or  $\hat{n}'$ ) is zero.

Thus

$$E(A^2\gamma') = S(\hat{S}' - \omega_c S) - \hat{S}(S' - \omega_c \hat{S}) \equiv B^2\beta'.$$

This proves Lemma 3.1(ii). Part (i) of Lemma 3.1 is proved similarly.

The analysis of the time-averaged quartic expressions of Section 3.2 is more tedious, but just as elementary. As instance, we will prove part (iii) with

$$c=0, \text{ viz., } E(2 A^2 \cdot A^2 \gamma' - A^4 \gamma') = 2 B^2 \cdot B^2 \beta' - B^4 \beta'.$$

Using equations (2) and the definition  $\gamma = \theta - \omega_c t$ ,

$$A^4\gamma' = (\hat{r}^2 + \hat{r}'^2)(\hat{r}(\hat{r}' - \omega_c \hat{r}) - \hat{r}'(\hat{r} + \omega_c \hat{r}')) = f_1(\hat{r}) - f_2(\hat{r}'),$$

where for notational convenience we define

$$f_1(\hat{r}) = (\hat{r}^2 + \hat{r}'^2)(\hat{r}(\hat{r}' - \omega_c \hat{r})); \quad f_2(\hat{r}') = (\hat{r}^2 + \hat{r}'^2)(\hat{r}'(\hat{r} + \omega_c \hat{r}')).$$

Thus

$$\begin{aligned}
 f_1(r) &= (S^2 + 2Sn + n^2 + \hat{S}^2 + 2\hat{S}n + \hat{n}^2) (\hat{S}\hat{S}' + \hat{S}n' + n\hat{S}' + n\hat{n}' - \omega_c (S^2 + 2Sn + n^2)) \\
 &= (S^2 + \hat{S}^2) (\hat{S}(\hat{S}' - \omega_c S)) + (S^2 + \hat{S}^2) (\hat{n}n' - \omega_c n^2) + (n^2 + \hat{n}^2) \hat{S}(\hat{S}' - \omega_c S) \\
 &\quad + (2Sn + 2\hat{S}n) (n(\hat{S}' - 2\omega_c S) + \hat{n}'S) + (n^2 + \hat{n}^2) n(\hat{n}' - \omega_c n) \\
 &\quad + \text{terms linear or cubic in noise.}
 \end{aligned}$$

Using equations (II.2) and (2), we get

$$\begin{aligned}
 E(f_1(r)) &= (S^2 + \hat{S}^2) (\hat{S}(\hat{S}' - \omega_c S) + (\omega_c \sigma^2 - \omega_c \sigma^2)) + 2\sigma^2 \hat{S}(\hat{S}' - \omega_c S) \\
 &\quad + 2S(\hat{S}' - 2\omega_c S)\sigma^2 + 2S^2\omega_c \sigma^2 + 4\sigma^2(\omega_c \sigma^2 - \omega_c \sigma^2) \\
 &= (S^2 + \hat{S}^2) \hat{S}(\hat{S}' - \omega_c S) + 4\sigma^2 \hat{S}(\hat{S}' - \omega_c S) = f_1(S) + 4\sigma^2 \hat{S}(\hat{S}' - \omega_c S).
 \end{aligned}$$

Similarly,

$$E(f_2(r)) = f_2(\hat{S}) + 4\sigma^2 \hat{S}(\hat{S}' + \omega_c \hat{S}).$$

Thus

$$\begin{aligned}
 E(A^h \gamma') &\equiv E(f_1(r) - f_2(r)) \\
 &= f_1(S) - f_2(\hat{S}) + 4\sigma^2 (\hat{S}(\hat{S}' - \omega_c S) - \hat{S}(\hat{S}' + \omega_c \hat{S})) = B^h \beta' + 4\sigma^2 B^2 \beta'.
 \end{aligned} \tag{II.4}$$

Recall that, given  $n$  samples  $X(t), \dots, X(nt)$  of a random process  $X$ , we have defined  $\bar{X} = (1/n) \sum_{j=1}^n X(jt)$ . If  $X$  and  $Y$  are uncorrelated at different sample times,

$$\begin{aligned}
E(\bar{X} \cdot \bar{Y}) &= (1/n^2) \left( \sum_{j \neq k} E(X(jt))E(Y(kt)) + \sum_j E(X(jt)Y(jt)) \right) \\
&= \overline{E(X) \cdot E(Y)} + \frac{1}{n} (\overline{E(XY)} - \overline{E(X)E(Y)}). \quad (II.5)
\end{aligned}$$

Equation (II.5) is valid for the case  $X = A^2$ ,  $Y = A^2 \gamma'$  because of our assumptions about the noise. With equations (4), (II.4) and (II.5), and Lemma 3.1(1), we have

$$\begin{aligned}
\overline{E(A^4 \gamma' - 2A^2 \cdot A^2 \gamma')} &= \overline{E(A^4 \gamma')} - 2 \overline{E(A^2) E(A^2 \gamma')} - (2/n) (\overline{E(A^4 \gamma')} - \overline{E(A^2)E(A^2 \gamma')}) \\
&= \overline{B^4 \beta' + 4\sigma^2 B^2 \beta'} - 2 \overline{(B^2 + 2\sigma^2) B^2 \beta'} - (2/n) (\overline{2\sigma^2 B^2 \beta'}) \\
&= \overline{B^4 \beta'} - 2 \overline{B^2 \cdot B^2 \beta'} - (4/n) (\overline{\sigma^2 B^2 \beta'}).
\end{aligned}$$

We can neglect the last term if  $2B^2 \gg (4/n)\sigma^2$ , ie if  $n \gg 1/\text{SNR}$ .

### APPENDIX III

#### PROBABILITY DISTRIBUTION OF THE INSTANTANEOUS FREQUENCY

Assume a Gaussian white noise signal with a band-limited power spectrum symmetrically distributed about its centre frequency and quadrature components  $n_I = r \cos \theta$  and  $n_Q = r \sin \theta$ . Let  $p(\theta')$  be the probability density function of the instantaneous angular frequency of this signal about its centre frequency.

Suppose  $\sigma^2$  is the power of the noise, and  $\theta'^2$  that of its time derivative.

Then (reference 7),

$$p(\theta') = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(r, \theta, r', \theta') d\theta dr dr',$$

where  $p(r, \theta, r', \theta')$  can be determined from

$$p(n_Q, n_I, n'_Q, n'_I) = (4\pi^2 \sigma^2 \theta'^2)^{-1} \exp(-(n_Q^2 + n_I^2)/2\sigma^2) \exp(-(n'^2_Q + n'^2_I)/2\theta'^2).$$

Specifically (cf equation (I.7) to (I.8)),

$$p(\theta') = (4\pi^2 \sigma^2 \theta'^2)^{-1} \int_0^\infty \int_0^\infty r^2 \exp(-r^2/2\sigma^2) \cdot \exp(-(r'^2 + r^2 \theta'^2)/2\theta'^2) \cdot d\theta \cdot dr \cdot dr',$$

so that

$$\begin{aligned} p(\theta') &\propto \int_0^\infty r^2 \exp(-r^2(1+\theta'^2 \cdot \sigma^2/\theta'^2)/2\sigma^2) \cdot dr \\ &\propto (1+\theta'^2 \cdot (\sigma^2/\theta'^2))^{-3/2}. \end{aligned}$$

Note that  $\sigma^2/\theta'^2$  is a function of the power spectrum shape and varies as  $B^{-2}$  with the bandwidth B.

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One of the important variables to be determined when monitoring unknown radio transmissions is the modulation type. Automatic recognition procedures based on time domain parameters additional to the standard envelope and instantaneous frequency are proposed. Initial simulations which incorporate Gaussian noise perturbation and centre frequency and bandwidth mismatch support the theoretical arguments that the new parameters provide better modulation type discrimination in noisy conditions.

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